

Thermodynamics of Deconfined Matter at Finite Chemical Potential in a Quasiparticle Description

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An effective quasiparticle description of the thermodynamics of deconfined matter, compatible with both finite-temperature lattice data and the perturbative limit, is generalized to finite chemical potential. Within this approach, the available 4-flavor lattice equation of state is extended to finite baryon density, and implications for cold, charge-neutral deconfined matter in β -equilibrium in compact stars are considered.

Key Words: quark-gluon plasma, deconfinement, equation of state, quasiparticle

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The equation of state (EoS) represents an important interrelation of state variables describing matter in local thermodynamical equilibrium. All microscopic characteristics are integrated out and only the macroscopic response to changes of the state variables as e.g., the temperature T and the chemical potential μ are retained. Via the Gibbs equation, $e = Ts + \mu n - p$, the energy density e is related to the entropy and particle densities, $s = \partial p / \partial T$ and $n = \partial p / \partial \mu$, respectively, and the pressure p . The thermodynamical potential $p(T, \mu)$ thus provides all information needed to evaluate, e.g., sequences of stellar equilibrium configurations by means of the Tolman-Oppenheimer-Volkov equation, or the evolution of the universe via Friedman's equations, or the dynamics of heavy-ion collisions within the framework of relativistic Euler equations. In the examples mentioned not only excited hadron matter is of relevance, rather, at sufficiently high density or temperature, a plasma state of deconfined quarks and gluons is the central issue.

Quantum Chromodynamics (QCD) is nowadays the generally accepted fundamental theory of interacting quarks and gluons. The challenge, therefore, consists in the derivation of the EoS of deconfined matter directly from QCD. In first attempts [1] the EoS of cold quark matter was derived perturbatively up to order $\mathcal{O}(g^4)$ in the coupling g . Finite temperatures have been considered [2], where the perturbative expansion is extended up to the order $\mathcal{O}(g^5)$. However, in the physically relevant region the coupling is large, so perturbative methods seem basically to fail and, consequently, nonperturbative evaluations are needed. Present lattice QCD calculations can accomplish this task, and indeed the EoS of the pure gluon plasma [3] and of systems containing two or four light dynamical quark flavors [4,5] are known at finite temperature, and simulations with physical quark masses may be available in the near future.

As a matter of fact, current QCD lattice calculations are restricted to the chemical potential $\mu = 0$ while a more detailed understanding of, e.g., the structure of quark cores in massive neutron stars, the baryon contrast prior to cosmic confinement, or the evolution of the baryon charge in the midrapidity region of central heavy-ion collisions requires the EoS at vanishing net quark density. Here we are going to develop an EoS, based on a phenomenological quasiparticle model, which extrapolates available lattice results at $\mu = 0$ to finite values of μ (even at $T = 0$) and at the same time interpolates smoothly to the asymptotic regime of QCD at $T \rightarrow \infty$ and $\mu \rightarrow \infty$, with the strangeness degree of freedom properly included. With our model we focus on deconfined matter and do not touch upon problems of confined matter or the deconfinement transition itself; the latter might be covered via usual *ad hoc* procedures.

We consider an $SU(N_c)$ plasma of gluons, with $N_c = 3$ for QCD, and N_f quark flavors in thermodynamic equilibrium. Within the approach outlined below, the interacting plasma is described in terms of a quasiparticle system, a picture arising asymptotically from the in-medium properties of the constituents of the plasma. For thermal momenta $k \geq \mathcal{O}(T)$ the relevant modes, transversal gluons and quark particle excitations, propagate predominantly on-shell with dispersion relations $\omega_i^2(k) \approx m_i^2 + k^2$ and

$$m_i^2 = m_{0i}^2 + \Pi_i^*, \quad (1)$$

while the longitudinal gluonic and helicity flipped quark states are essentially unpopulated [6]. Neglecting sub-leading effects, Π_i^* are the leading order on-shell self-energies of parton species i . Depending on the coupling G^2 , the temperature and chemical potential as well as the rest mass m_{0i} (the latter vanishing for gluons), the Π_i^* are given by the asymptotic values of the hard thermal/density loop selfenergies [6],

$$\begin{aligned} \Pi_q^* &= 2 \omega_{q0} (m_0 + \omega_{q0}), \quad \omega_{q0}^2 = \frac{N_c^2 - 1}{16 N_c} \left[T^2 + \frac{\mu_q^2}{\pi^2} \right] G^2, \\ \Pi_g^* &= \frac{1}{6} \left[\left(N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2\pi^2} \sum_q \mu_q^2 \right] G^2. \end{aligned} \quad (2)$$

Generalizing the approach of ref. [7] to a finite chemical potential μ controlling a conserved particle number, the pressure of the system can be decomposed into the contributions p_j of the quasiparticles, and their mean field interaction B ,

$$p(T, \mu; m_{0i}^2) = \sum_j p_j(T, \mu_j(\mu); m_j^2) - B(\Pi_i^*), \quad (3)$$

where formally p_j is the pressure of an ideal gas of bosons or fermions with state dependent effective masses (1,2). By the stationarity of the thermodynamic potential $\Omega = -pV$ under functional variation of the selfenergies [8], which in the present approach simplifies to $\partial p / \partial \Pi_j^* = 0$, B is related to the quasiparticle masses,

$$\frac{\partial B}{\partial \Pi_j^*} = \frac{\partial p_j(T, \mu_j; m_j^2)}{\partial m_j^2}, \quad (4)$$

which implies that the entropy and particle densities are given by the sum of the quasiparticle contributions

$$s_j = \left. \frac{\partial p_j(T, \mu_j; m_j^2)}{\partial T} \right|_{m_j^2}, \quad n_j = \left. \frac{\partial p_j(T, \mu_j; m_j^2)}{\partial \mu_j} \right|_{m_j^2}. \quad (5)$$

The quasiparticle approach represents an effective resummation of the leading-order thermal contributions [9]. Hence, it is expected to be an appropriate framework as long as the spectral properties of the relevant plasma excitations do not differ qualitatively from their asymptotic form. In the hot ϕ^4 theory, for which an equivalent thermodynamic quasiparticle description was derived by resumming all tadpole diagrams [10], this assumption was shown to be fulfilled also at larger values of the coupling by resumming the propagator beyond 1-loop order [11]. Corroborating the assumption for QCD thermodynamics at $\mu = 0$, the quasiparticle approach was found to be in a remarkably good agreement with lattice data even close to the confinement region [12–14], with the effective coupling G^2 in eq. (2) parameterized by

$$G^2(T, \mu = 0) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left(\frac{T + T_s}{T_c/\lambda} \right)^2}, \quad (6)$$

interpolating smoothly to the asymptotic QCD limit.

For finite values of μ , we observe that due to the stationary property (4) the dependence of the function B on the state variables is determined by

$$\frac{\partial B}{\partial T} = \sum_j \frac{\partial p_j}{\partial m_j^2} \frac{\partial \Pi_j^*}{\partial T} \equiv B_T, \quad \frac{\partial B}{\partial \mu} = \sum_j \frac{\partial p_j}{\partial m_j^2} \frac{\partial \Pi_j^*}{\partial \mu} \equiv B_\mu. \quad (7)$$

The pressure (3) and thus the function B being a potential of the state variables T and μ , the functions B_T and B_μ have to respect the integrability condition

$$\frac{\partial B_T}{\partial \mu} - \frac{\partial B_\mu}{\partial T} = \sum_j \left[\frac{\partial n_j}{\partial m_j^2} \frac{\partial \Pi_j^*}{\partial T} - \frac{\partial s_j}{\partial m_j^2} \frac{\partial \Pi_j^*}{\partial \mu} \right] = 0. \quad (8)$$

In the selfenergies, eq. (2), the effective coupling itself is a function of the state variables, so eq. (8) represents a

first order partial differential equation for $G^2(T, \mu)$. With $G^2(T, \mu)$ given, e.g., at $\mu = 0$ by lattice data, this flow equation determines the effective coupling and hence by eqs. (1-3) and (7) the EoS of the plasma at finite temperature and chemical potential.

As a first example let us now apply the quasiparticle approach to the EoS of the QCD plasma with $N_f = 4$ light flavors, which is numerically known for $\mu = 0$ in a restricted interval of T . From a fit (cf. fig. 1 in ref. [13]) of the lattice data [5] we obtain the parameters $\lambda = 6.7$ and $T_s/T_c = -0.81$ of the effective coupling (6), and the gluon and quark quasiparticle degrees of freedom $d_g = 20.6$ and $d_q = \frac{4N_c N_f}{2(N_c^2 - 1)} d_g$. The large value of the parameter T_s/T_c points to the distinct nonperturbative behavior at $T \rightarrow T_c$. We note that the ratio of the fitted value of d_g to the expected value of 16 is of the same order of magnitude as finite lattice size effects in the interaction free limit.

With this at hand we extrapolate the lattice data to finite values of $\mu_q = \mu$, corresponding to a finite net quark density. Eq. (8) is a quasilinear partial differential equation of the form

$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b, \quad (9)$$

with the coefficients $a_{T,\mu}$ and b depending on T, μ and G^2 , which can be solved by the method of characteristics. It is instructive to consider first the asymptotic limit, $m_i^2/T^2 \sim G^2 \rightarrow g^2 \rightarrow 0$, where (9) reduces to

$$\pi^2 \left(cT^2 + \frac{\mu^2}{\pi^2} \right) \frac{1}{\mu} \frac{\partial g^2}{\partial \mu} - \left(T^2 + \frac{\mu^2}{\pi^2} \right) \frac{1}{T} \frac{\partial g^2}{\partial T} = 0, \quad (10)$$

with $c = (4N_c + 5N_f)/(9N_f)$. This equation yields $g^2 = \text{const}$ along the characteristics given by $cT^4 + 2T^2(\mu/\pi)^2 + (\mu/\pi)^4 = \text{const}$. As a result of this elliptic flow, the renormalization scale T_c/λ of the effective coupling $g^2(T, 0)$ determines the scale $T_c \pi c^{1/4}/\lambda$ of $g^2(0, \mu)$, and the pressure (1-3) coincides with the perturbative QCD expression up to the order $\mathcal{O}(g^2)$.

The flow (9) of the effective coupling, with $G^2(T, 0)$ obtained from the lattice data referred to above, is shown in fig. 1. The characteristics related to larger temperatures resemble their asymptotic form, and the coupling displayed in fig. 2 decreases asymptotically as expected. For smaller values of T and μ , the pronounced increase of G^2 is considered to be a sign of the vicinity of the phase transition. Indicated by intersecting characteristics, the solution of the flow equation is non-unique in a certain low-temperature region. In this region, however, the pressure turns out to be negative, so the ambiguity is of no physical relevance. To obtain the pressure, which is available in tabular form [15], the function $B(T, \mu)$ can be computed along the characteristics using eq. (4). Hereby, the required function $B(T, \mu = 0)$ is determined by the first equation of (7) up to an integration constant

$B_0 = B(T_c, 0)$ which is fixed by equating the quasiparticle pressure and the lattice data at T_c . For chemical potentials $\mu \sim 2.5 T_c$ and small temperatures, in a region of the phase space corresponding to characteristics emanating from the vicinity of the point $(T_c, \mu = 0)$, the pressure becomes negative. This region of instability provides a lower boundary for the value of the confining chemical potential $\mu_c \geq 3.4 T_c$ at $T = 0$. As evident from fig. 1, this boundary also excludes the region of non-unique flow. Hence, the quasiparticle model is intrinsically consistent and therefore considered to provide a realistic extension of the EoS obtained by lattice calculations to $\mu \neq 0$ even near the confinement transition.

To conclude this example we remark that by the Feynman-Hellmann relation the quasiparticle pressure (3) leads to a chiral condensate $\langle \bar{\psi}\psi \rangle \sim m_{0q}$, so for massless quark flavors the restoration of chiral symmetry in the deconfined phase is inherent in the approach.

Thermodynamic lattice simulations of QCD with physical quark current masses, $m_{0u,d} \sim 0$ for the light flavors and $m_{0s} \sim 150$ MeV for strange quarks are still lacking, so the parameters of the quasiparticle description cannot be fixed at finite temperature to extend the physically relevant EoS to $\mu \neq 0$. However, a trial EoS $p^{\text{trial}}(T, \mu = 0)$ can be constructed by reasonable choices of the model parameters until they may be specified by precise lattice data. Due to the stationarity property (4) of the thermodynamic potential, $p^{\text{trial}}(T, \mu = 0)$ is expected to possess only a weak dependence on the parameters. The model parameters are restricted to match the hadronic pressure p^{had} at the confinement temperature which we assume to be $T_c = 150$ MeV. The uncertainty of $p^{\text{had}}(T_c, 0) = 3.1 \cdot 10^8 \text{ MeV}^4$ as obtained by a hadron resonance gas model can be absorbed into the integration constant B_0 , which we vary independently beside the scaling parameter λ in eq. (6). Being related to the QCD scale parameter Λ_{QCD} , λ is expected to be a slowly increasing function of the number of flavors; considering $3 \leq \lambda \leq 9$ we cover the values $\lambda^{N_f=0} = 4.2$ as obtained from the pure gluon lattice data [3] and $\lambda^{N_f=4} = 6.7$ in the example above.

With the aim to study implications for quark matter stars, we consider in the following a plasma of gluons, quarks and electrons in β -equilibrium maintained by the reactions $d, s \leftrightarrow u + e + \bar{\nu}_e$, which imply the relations $\mu_d = \mu_s = \mu_u + \mu_e \equiv \mu$ among the chemical potentials. The electron chemical potential μ_e as a function of T and μ is determined by the requirement of electrical charge neutrality. With the electron pressure p^e approximated by its free limit, the pressure $p = p^{\text{trial}} + p^e$ and the resulting energy density are shown for several values of the parameters λ and B_0 at $\mu = 0$ in the left panels of fig. 3. Already at temperatures slightly above T_c , the scaled energy density reaches a saturation-like behavior at some 90% of the asymptotic limit. This feature, known qualitatively from the lattice simulations [3–5], is to a large extent insensitive to the specific choice of the free parameters, while B_0 has a distinct impact on the latent heat.

At vanishing temperature, the resulting EoS is displayed in the right panels of fig. 3. In this case, the asymptotic values are approached more slowly due to the less rapid decrease of the effective coupling with increasing values of μ , similar to the $N_f = 4$ plasma.

With the resulting EoS $e(p)$, sequences of hydrostatic equilibrium configurations of cold stars can be calculated by the Tolman-Oppenheimer-Volkov (TOV) equation (cf. eq. (2.212) of ref. [16]). For energy densities up to several times the nuclear density and at temperatures less than some 10 MeV, the EoS of β -stable quark matter, as estimated by our quasiparticle approach can be parameterized by $e = 4\tilde{B} + \tilde{\alpha}p$. While the naive bag model EoS has $\tilde{\alpha} = 3$, our quasiparticle EoS, with the considered choices of the model parameters, yields $3.1 \leq \tilde{\alpha} \leq 4.5$, indicating the nontrivial nature of the interaction. For parameters $\lambda \geq 5$, values of $\tilde{B}^{1/4} \geq 200$ MeV are found, while only the extreme choice of $\lambda = 3$ allows $\tilde{B}^{1/4}$ as small as 180 MeV. However, the value of \tilde{B} , i. e., the energy density at small pressure, has a strong impact on the star's mass and radius, obtained by integrating the TOV equation. Compact quark stars of mass $M \geq M_\odot$, e. g., could only exist for values of $\tilde{B}^{1/4} \leq 180 \dots 200$ MeV, depending marginally on $\tilde{\alpha}$. We hence conclude that compact stars with typical masses $\sim M_\odot$ and radii ~ 10 km are unlikely to be composed purely of deconfined matter in β -equilibrium, irrespective of the uncertainty of the model parameters of our approach.

In summary we have generalized a thermodynamic quasiparticle description of deconfined matter to finite chemical potential μ not accessible by present lattice calculations. Of central importance to the model is the effective coupling $G^2(T, \mu)$ which can be obtained at $\mu = 0$ from available lattice data, proving at the same time the applicability of the effective description even close to the confinement transition. At finite chemical potential, G^2 is determined by a flow equation resulting from the general requirement of integrability. By the flow of the effective coupling, the basic features of the EoS at $\mu = 0$, namely the nonperturbative behavior near confinement and the asymptotics, are mapped into the T - μ plane as exemplified by the $N_f = 4$ flavor system studied on the lattice. An important consequence of the quasiparticle approach is the relation of the critical values of temperature and chemical potential. For deconfinement matter with physical quark masses, this fact leads to the implication that compact stars composed purely of β -stable deconfined matter may be less massive and, hence, more distinct in the bulk properties to neutron stars than estimated by other approaches.

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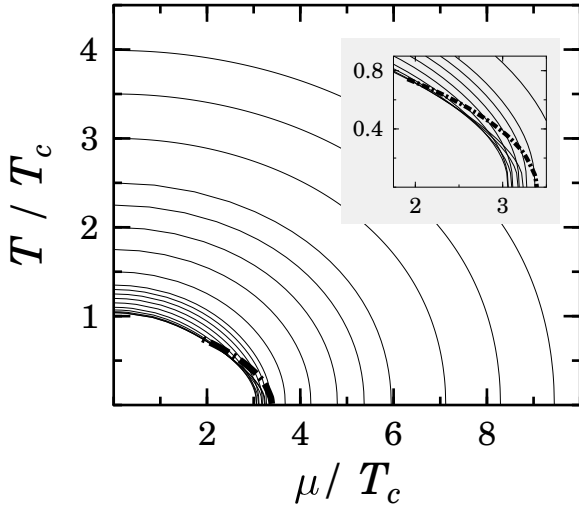


FIG. 1. The characteristics of the coupling flow equation (9) for the QCD plasma with $N_f = 4$ light flavors for which $G^2(T, \mu = 0)$ is obtained from lattice data [5]. At leading order the characteristics are curves of constant coupling strength. The pressure is negative in the region below the dash-dotted line, thus excluding the region of intersecting characteristics.

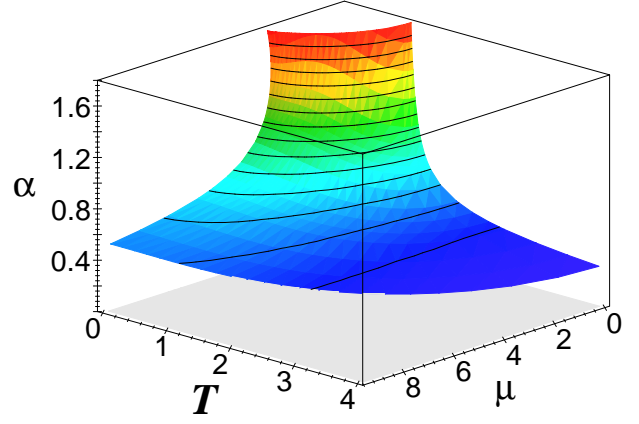


FIG. 2. The effective coupling strength $\alpha = G^2/(4\pi)$ as a function of μ and T for the $N_f = 4$ plasma in the chiral limit.

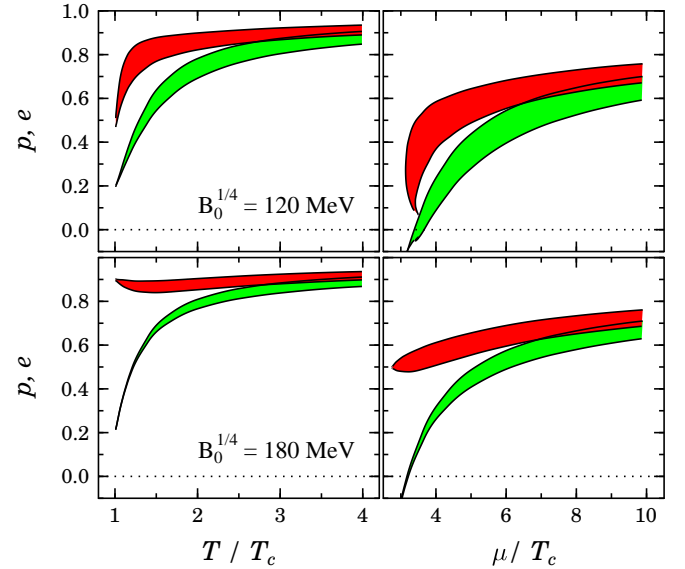


FIG. 3. Total pressure and energy density (lower and upper set of curves, respectively) of the charge-neutral, quark-gluon plasma in β -equilibrium, scaled by the values of the free limit. The panels on the left (right) show the EoS at $\mu = 0$ ($T = 0$), for values of the model parameter $3 \leq \lambda \leq 9$ (lower and upper line, respectively, of hatched area), and $B_0^{1/4} = 120$ MeV (top) and $B_0^{1/4} = 180$ MeV (bottom). Non-unique values of the energy only occur in the unphysical region, where $p < 0$.